Exercise 12

Find the differential of each function.

$$\text{(a)}\ \ y = \frac{1+2u}{1+3u} \qquad\qquad \text{(b)}\ \ y = \theta^2 \sin 2\theta$$

Solution

Part (a)

Compute the derivative of y.

$$\begin{aligned}
\frac{dy}{du} &= \frac{d}{du}\left(\frac{1+2u}{1+3u}\right) \\
&= \frac{\left[\dfrac{d}{du}(1+2u)\right](1+3u) - \left[\dfrac{d}{du}(1+3u)\right](1+2u)}{(1+3u)^2} \\
&= \frac{(2)(1+3u) - (3)(1+2u)}{(1+3u)^2} \\
&= \frac{-1}{(1+3u)^2} \\
&= -(1+3u)^{-2}
\end{aligned}$$

Therefore, the differential of $y = (1+2u)/(1+3u)$ is

$$dy = -(1+3u)^{-2}\,du.$$

Part (b)

Compute the derivative of y.

$$\begin{aligned}
\frac{dy}{d\theta} = \frac{d}{d\theta}(\theta^2 \sin 2\theta) &= \left[\frac{d}{d\theta}(\theta^2)\right]\sin 2\theta + \theta^2\left[\frac{d}{d\theta}(\sin 2\theta)\right] \\
&= (2\theta)\sin 2\theta + \theta^2\left[(\cos 2\theta)\cdot\frac{d}{d\theta}(2\theta)\right] \\
&= 2\theta \sin 2\theta + \theta^2[(\cos 2\theta)\cdot(2)] \\
&= 2\theta(\sin 2\theta + \theta\cos 2\theta)
\end{aligned}$$

Therefore, the differential of $y = \theta^2 \sin 2\theta$ is

$$dy = 2\theta(\sin 2\theta + \theta\cos 2\theta)\,d\theta.$$